The process of wealth accumulation in consideration of the path dependence theory

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Abstract
This paper analyses the process of wealth accumulation in consideration of the path dependence theory. Based on the theoretical foundations of Merton, Pareto and Bourdieu, the mechanisms of wealth accumulation will be analysed. Furthermore, these mechanisms, which are understood as direct and indirect network effects, will be formalized by the statistical Software R in form of different models. That makes it possible to include the analysed mechanisms step by step and observe their effects on the process of wealth accumulation and social inequality. Moreover, Piketty’s findings out of his work *Capital in the 21st century*, in particular the relationship between the rate of return to capital and the growth rate, will be included in the formalized models of wealth accumulation.

1 Introduction
The aim of this paper is the analysis of the process of wealth accumulation in consideration of the path dependence theory. To describe the phenomenon of resistant and manifest social inequality, the paper identifies mechanisms which have influenced the process of wealth accumulation. In this context the concept of path dependency is used, which shows the process of developing inequality in a theoretical way. The paper understands these mechanisms as direct and indirect network effects, which can be used as analytical instruments of a path depended development.

As a theoretical foundation of a direct network effect, the paper uses the concept of the “matthew effect“ ([Merton, 1968]), which contains the approach of cumulative advantages for high wealth. These cumulative advantages cause increased social inequality and wealth accumulation. Next Bourdieu’s capital theory will be used to analyse an indirect network effect. According to this theory, a large amount of economic capital leads to an easier accumulation of cultural and social capital. These two forms of capital can be furthermore used to accumulate economic capital, which leads finally to an indirect network effect of wealth accumulation. Moreover, Pareto’s theory of society will be analyzed to find a second indirect network
effect of wealth accumulation. According to Pareto, political power and economic capital need each other and lead to positive feedback loops. Therefore, political power leads to economic capital and vice versa. This supports the phenomenon of shadow banks, which are built from social elites and in addition cause matthew effects. Moreover, inspired by Piketty’s work *Capital in the 21st century*, this paper will look how the created models react when the rate of return to capital increases over the growth rate of the economy. The core of the following paper will be the development of a model which shows the analysed direct and indirect network effects. Based on an equation of Meade, the framework of the model will be developed, simulated and illustrated by graphs. In addition, direct and indirect network effects will be included step by step to simulate the process of wealth accumulation. This should create a better understanding how the process of wealth accumulation works and which mechanisms need to be taken into account when this process is discussed in the current debate of social inequality.

2 Path dependence

The theory of path dependence implies that events in the past influence the present and the future and can possible cause lock ins where technological standards and states of society cannot be changed any more. This process is set off by critical junctures and furthermore strengthened through direct and indirect network effects [Sydow/Schreyögg/Koch, 2009, p.690]. According to Liebowitz and Margolis a network effect can be defined as a “circumstance in which the net value of an action [...] is affected by the number of agents taking equivalent actions.” [Liebowitz/Margolis, 1994, p.135],[Liebowitz/Margolis, 1994, p.135]. Figure 1 shows the possible development of a path depended process. Phase 1, the preformation phase, is characterized by an open situation with no significantly restricted scope of action. The border crossing point to phase 2, the formation phase, is marked with the occurrence of a critical juncture, which represents a decision or an action that amounts to a trigger for the further development. At this point of the process, direct and indirect network effects take place, which are causing cumulative and
self-reinforcing advantages. This development can finally lead to a further restriction of the scope and possible cause a lock in, which is represented by phase 3 [Sydow/Schreyögg/Koch, 2009, p.692ff].

Figure 1: Path dependence [Sydow/Schreyögg/Koch, 2009, p.692]

This paper takes its focus on the role of direct and indirect network effects, which will be explained theoretically and formalized in the following sections. Direct network effects typically occur in a physical two-way communication network, whereas indirect network effects are found in networks with compatible devices or systems [Page/Lopatka, 1999, p. 954ff].

3 Indirect network effect: Matthew-effect

3.1 Theory: Matthew-effect

The Matthew-effect was firstly described by Merton on the basis of the reward system in science. Merton showed that eminent scientists get disproportionately great credit for their contributions to science, whereas relatively unknown scientists tend to get disproportionately little credit for comparable contributions [Merton, 1968, p.1ff]. Therefore, success depends not only on
contemporary performance, but on performance in the past. As a result, the Matthew-effect causes that initial differences at the beginning of a process are getting larger in the long term [Lutter, 2012, p.435f.]. Rigney showed this phenomenon on the basis of the effect of interest rates. Figure 2 shows three different initial conditions. Line A represents a person with $1000, line B a person with $100 and line C a person with a debt of $1000. All three asset stocks are running 10 years and have an annual interest rate of 10%. Owing to the interest rate of 10%, after 10 years the differences between person A, B and C are bigger than at the beginning. While at year 0 the difference between line A and line B was $900, it has grown to $2335 in year 10. The same effect can be observed between line A and line C where the difference has grown from $2000 in year 0 to $5188 in year 10 [Rigney, 2010, p.11].

![Graph showing compound interest and compound debt over 10 years](image)

**Figure 2:** Matthew-effect (Rigney, 2010, p.11)

With the help of this example it was possible to show that the Matthew-effect causes, due to its cumulative advantages, that small initial differences in wealth get larger differences in the future. To formalize this direct network effect a formal model will be developed. For this purpose the Software R will be used [R Development Core Team, 2014].
3.2 Model 1

The first formal model includes no direct or indirect network effects and should only provide a foundation for the analysed mechanisms of wealth accumulation. Model 1 has the following assumptions: There will be 500 individuals and 200 simulated rounds. Each individual gets a normal distributed earned income with mean 5 and a standard deviation of 1. This income is calculated at round 1, will be paid out each round and stays the same for all 200 rounds. The consumption rate of each individual is 90% of the earned income. Each individual has an initial asset of 10. Figure 3 shows that there is no considerable difference between the individuals in the distribution of wealth after 200 rounds. To analyse the effects on the process of wealth accumulation various indicators have been calculated in Table 1. These indicators will be used to compare the following models with each other and show how the developed direct and indirect network effects influence the process of wealth accumulation. Therefore, the 20/20 and 10/10 ratio, which simply shows the ratio between the highest 20% respectively 10% and the lowest 20% respectively 10% of wealth owners, is calculated. The higher the ratio, the more social inequality can be observed. Furthermore, the gini-coefficient is used to show the effects of the analysed direct and indirect network effects. This coefficient would reach a value of 0 if there is a minimal and a value of 1 if there is a maximal concentration of wealth [Quatember, 2008, p.60]. In addition, Table 1 shows the minimal and maximal wealth an individual was able to reach after 200 rounds. Moreover, the total wealth of all individuals after 200 rounds is shown.
3.3 Model 2

To include the analysed direct network effect, model 1 will be extended by the following assumptions: A normal distributed interest rate with a mean of 3% and a standard deviation of 0.1/100 will be established. Furthermore, an adjusted savings rate will be introduced: if the wealth of an individual is equal or larger than 1.2 times the average wealth, the consumption rate
is 75% of the earned income and if the wealth of an individual is equal or larger than 1.5 times the average wealth, the consumption rate is 65% of the earned income. Therefore individuals with a high amount of wealth are able to increase their wealth faster than individuals with a low amount of wealth. Moreover, the growth rate of GDP is determined by 3% per round. In addition, an accounting identity is used to calculate the wealth of an individual.

\[ W_t = W_{t-1} + E_t + r_t \cdot W_{t-1} - C_t + I_t \]

This simple framework should provide the foundation to analyse the process of wealth accumulation over time. Variable \( W_t \) represents the accumulated wealth in round \( t \), \( W_{t-1} \) the accumulated wealth in round \( t-1 \), \( E_t \) the earned income in round \( t \), \( r_t \cdot W_{t-1} \) the capital income in round \( t \), \( C_t \) the consumption in round \( t \) and \( I_t \) the inheritances in round \( t \). The last variable will be absorbed by the initial wealth of 10 [Davies/Shorrocks, 2000, p.610]. Furthermore, another accounting identity is used to calculate the growth rate of earned income. As the interest rate for capital \( r_c \), the growth rate of GDP \( g \), the sum of earned income \( \sum EI_{t-1} \cdot (1 + r_t) \) in period \( t-1 \) and the sum of capital income \( \sum W_{t-1} \cdot r_c \) in period \( t-1 \) are given, the growth rate of earned income \( r_e \) can be calculated easily by transforming the following equation:

\[ GDP_{t-1} \cdot (1 + g) = \sum EI_{t-1} \cdot (1 + r_e) + \sum W_{t-1} \cdot r_c \]

\[ GDP_t = \sum W_{t-1} \cdot r_c = \sum EI_{t-1} \cdot (1 + r_e) \]

\[ \frac{GDP_t - \sum W_{t-1} \cdot r_c}{\sum EI_{t-1}} = 1 + r_e \]

\[ r_e = \frac{GDP_t - \sum W_{t-1} \cdot r_c}{\sum EI_{t-1}} - 1 \]
By taking the new assumptions into account, model 2 can be simulated and observed in Figure 4. Owing to the consideration of the analysed direct network effect the distribution of wealth after 200 rounds has changed. Now a small elite of individuals has the chance to accumulate significantly more wealth than the rest. Furthermore, the composition of GDP shows that the importance of capital income is growing per time. This is underlined by the development of Piketty’s Beta, which represents the ratio between total wealth and GDP and is an indicator for the importance of capital in the process of wealth accumulation. The higher this indicator the higher is the importance of capital in the process of wealth accumulation [Piketty, 2014, p.50f.]. Moreover, Figure 4 shows the development of total wealth. The indicators in Table 2 show that, owing to the implementation of the direct network effect, social inequality has increased. The 20/20, respectively 10/10, ratio has grown from 1.676/2.269 in model 1 to 2.269/3.199 in model 2. Moreover, the gini-coefficient, which has increased from 0.102 in model 1 to 0.171 in model 2, highlights the increase of social inequality. Therefore, it is obvious that, due to the implementation of the direct network effect, social inequality has increased and the process of wealth accumulation has been influenced.
The distribution of wealth after 200 rounds

Figure 4: Own Illustration

<table>
<thead>
<tr>
<th></th>
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</tr>
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<tbody>
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</tr>
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<td>Gini-coefficient</td>
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</table>

Table 2: Own calculations
The next section will analyse indirect network effects of the process of wealth accumulation. For this purpose the capital theory of Bourdieu and Pareto’s theory of society will be objects of a closer examination.

4 Indirect network effect: Bourdieu

4.1 Theory: Bourdieu

According to Bourdieu, capital causes an immanent regularity of the social world and determines the chances of success an individual has. In addition, capital needs time to be accumulated and has a potential capacity to reproduce itself in identical or expanded form. Furthermore, capital contains a tendency to persist in its being and leads to the fact that everything is not equally possible or impossible. Therefore, it is necessary to know that the structure of the distribution of the different types and subtypes of capital represents the immanent structure of the social world. Moreover, Bourdieu states that economic theory reduces the universe of exchanges to mercantile exchanges. Other forms of exchanges are disinterested and explicitly defined as non-economic. As a result, economic theory cannot take the complex structure of the real world and all its forms of capital and interactions into account. Moreover, Bourdieu mentions that capital can present itself in three fundamental forms: First economic capital, which is directly convertible into money and can be institutionalized in the form of property. Cultural capital, which can be institutionalized in the form of educational qualifications and social capital, which can be institutionalized in the form of a title of nobility. Bourdieu states that cultural and economic capital are convertible into money [Bourdieu, 1983, p.83ff.]. To understand that economic and social, respectively cultural capital are compatible systems, which create cumulative advantages for individuals, a further explanation is needed.

Cultural capital

Cultural capital can exist in three forms. The first form is the embodied state, which reflects the form of long-lasting dispositions of the mind and
body [Bourdieu, 1983, p.84]. According to Krenz, the accumulation of this form of capital can only happen through the individual itself [Krenz, 2008, p.7]. Moreover, there is the objectified state, which is represented by cultural goods as pictures, instruments and books. The institutionalized state can be expressed in the form of educational qualifications, such as a college degree. According to Bourdieu, economists are not capable to understand the impact of cultural capital on the process of wealth accumulation because they only take monetary investments and profits, or those directly convertible into money, into account. For this reason, they forget that the scholastic yield from educational action depends not only on the cost of study but also on the cultural capital previously invested by the family [Bourdieu, 1983, p.84f.]. Therefore, it is obvious that economic and cultural capital are two complement systems, which benefit from each other through positive feedback effects.

Social capital
According to Bourdieu, social capital is „the aggregate of the actual or potential resources which are linked to possession of a durable network of more or less institutionalized relationship of mutual acquaintance and recognition—or in other words, to membership in a group“ [Bourdieu, 1983, p.88]. Moreover, Bourdieu states that the reproduction of social capital needs time and effort and is therefore direct or indirect economic capital [Bourdieu, 1983, p.88ff.].

The analysis of Bourdieu’s concept of cultural and social capital shows that a high amount of economic capital can lead to a high amount of social and cultural capital and vice versa. Therefore, an indirect network effect can be observed. Model 3 will take the impact of social and cultural capital into account and shows how these two forms of capital can affect the process of wealth accumulation.
4.2 Model 3

To include the analysed indirect network effect, model 2 will be extended by the following assumption: If the wealth of an individual is larger than 1.1 times the average wealth, the individual receives an earned income, which is 20% higher than before.

Figure 5 shows that the distribution of wealth after 200 rounds has changed. Compared to Figure 4 the distribution of wealth indicates that social elites were able to put a distance between them and the rest of society. Furthermore, capital income has increased and surpasses earned income. Moreover, Table 3 shows that the indicators have changed and social inequality has increased. The 20/20, respectively 10/10, ratio has increased from 2.269/3.199 in model 2 to 3.161/5.221 in model 3. Furthermore, the gini-coefficient has increased from 0.171 in model 2 to 0.254 in model 3 and indicates an increase in social inequality. The range between the minimum and maximum wealth has increased as well. The minimum wealth decreased from 15,137 in model 2 to 11,305 in model 3, whereas the maximum wealth increased from 101,876 from model 2 to 186,072 in model 3. The results show that the implementation of the indirect network effect has an impact on the process of wealth accumulation and that social inequality has increased.
Figure 5: Own Illustration

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Table 3: Own calculations
4.3 Theory: Pareto

The second theoretic foundation for the development of an indirect network effect will be a study from the Italian sociologist and economist Vilfredo Pareto. Figure 6 shows the results of his research on income distribution, for which he collected data from England, Italy, Germany, Paris and Peru. Pareto called the distribution in Figure 6 "social pyramid", which shows that most people have similar income, whereas only a small part of the population has a very low or high income [Persky, 1992, p.182ff.]. According to Pareto, the distribution of income does not change over time and is not resulting out by pure chance, but depends on the distribution of physiological and psychological characteristics of human beings [Pareto, 1975, p.112ff.]. Owing to this distribution Pareto divided the society into different classes that have a different chance to accumulate wealth and power [Riener, 1995, p.62]. According to Pareto, individuals who have the most wealth also have the most political and economical influence in a society which help them to reproduce their favourable economic situation [Pareto, 1975, p.113]. Therefore, it is obvious that wealth and political power overlap and form a strong social elite. Moreover, Pareto stated that social elites are in a constant competition with competitors who are trying to reach political and economical power. As a result, social elites need to use political institutions to install legal systems that protect them from their competitors [Pareto, 1975, p.132]. According to Pareto’s conclusions social elites can use their power to design legal systems, which help them to reproduce the situation of social inequality by assuring themselves advantages in the game of power and in the process of wealth accumulation. Therefore, a second indirect network effect can be observed because a system of two compatible systems, power and wealth, is apparent. It becomes obvious that political power and economic capital cause positive feedback loops and as a result political power leads to economic capital and vice versa. Social elites use their political influence, which overlaps with economic wealth, to build a legal system that enables them to reproduce social inequality.
To formalize this indirect network effect the phenomenon of shadow banks will be added to model 2. Shadow banks represent legal systems which are used from social elites to reproduce social inequality. The argument of the inequality of returns on capital is supported by Piketty, who showed that already high wealth owners have a higher average real growth rate of capital than the rest of society.

According to the European Commission, shadow banks have a strong impact on society and the economy. Therefore, shadow banks held about $46 billion in the year 2010 which is about 25-30% of the global financial system and 50% of all bank resources [European Commission, 2012, p.5]. According to Liebert, Ötsch and Troost, the shadow banking system held $67 billion which represented about 86% of the worldwide GDP and 90% of the global financial securities in the year 2012 [Liebert/Ötsch/Troost, 2013, p.15]. These numbers underline the relevance of the shadow banking system and show that it is a global phenomenon. Moreover, Liebert, Ötsch and Troost argue that social elites have political influence which they use to prevent any interventions by the state and reforms which could reduce the phenomenon.
of shadow banking. In fact the debate takes its focus on the self-healing mechanism of the market, which is a result of the neoliberal paradigm in politics and economics, and therefore a political discussion cannot take place [Liebert/Ötsch/Troost, 2013, p.5f.]. However, figure 7 underlines the argument of the inequality of returns on capital. Between the year 1987 and 2013, the average real growth rate of wealth per adult was 2.1% whereas it was 6.8% for the top 1/(100 million) highest wealth owners [Piketty, 2014, p.435].

![The growth rate of top global wealth, 1987-2013](image)

**Figure 7:** The growth rate of top global wealth, 1987-2013 [Piketty, 2014, p.435]

Moreover, Piketty showed that the average real annual rate return on capital endowments of US universities is increasing with higher endowments. Piketty took US universities because they publish regular, reliable, and detailed reports of their endowments. Figure 8 shows two main conclusions. First, between 1980-2010 the endowments of US universities had been extremely high and second, the return increases rapidly with size of endowment. For endowments with less than $100 million the average real annual rate of return was 6.2%, whereas endowments higher than $1 billion had an average real annual rate of return of 8.8%. The top trio Harvard, Yale and Princeton even got an average real annual rate of return of 10.2%. There-
fore, it is obvious that the greater the endowment is, the greater is the return [Piketty, 2014, p.447f].

<table>
<thead>
<tr>
<th>The return on the capital endowments of US universities, 1980–2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average real annual rate of return (after deduction of inflation and all administrative costs and financial fees) (%)</td>
</tr>
<tr>
<td>All universities (850)</td>
</tr>
<tr>
<td>Harvard, Yale, and Princeton</td>
</tr>
<tr>
<td>Endowments higher than $1 billion (60)</td>
</tr>
<tr>
<td>Endowments between $500 million and 1 billion (66)</td>
</tr>
<tr>
<td>Endowments between $100 million and $500 million (216)</td>
</tr>
<tr>
<td>Endowments less than $100 million (498)</td>
</tr>
</tbody>
</table>

Note: Between 1980 and 2010, US universities earned an average real return of 8.2% on their capital endowments, and all the more so for higher endowments. All returns reported here are net of inflation (1.4% per year between 1980 and 2010) and of all administrative costs and financial fees.
Sources: See piketty.pse.ens.fr/capital21c.

Figure 8: Return on the capital endowments of US universities, 1980-2010 [Piketty, 2014, p.448]

4.4 Model 4

To include the presence of shadow banks, which cause a higher interest rate for already wealthy individuals, the following assumptions are included to model 2: If the wealth of an individual is larger than 1.5 times the average wealth, the interest rate will increase by 1 percentage point.

Owing to the implementation of shadow banks the distribution of wealth after 200 rounds has changed. Figure 7 shows that a social elite occurred which is even more elitist than in model 3. Furthermore, capital income has surpassed earned income and the composition of GDP has changed dramatically. In addition, the 20/20, respectively 10/10, ratio has increased from 2.269/3.199 in model 2 to 3.506/5.924 in model 4. Moreover, the gini-coefficient has increased from 0.171 in model 2 to 0.290 in model 4, which highlights the growth in social inequality. Furthermore, the maximum wealth has increased significantly from 101,876 in model 2 to 315,320 in model 4. Therefore, it is
obvious that the implementation of an indirect network effect has an impact on the process of wealth accumulation and has caused higher social inequality.

![Figure 9: Own Illustration](image-url)
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Table 4: Own calculations

5 Piketty’s implications to the process of wealth accumulation

5.1 Analysis

To discuss the process of wealth accumulation with a wider foundation Piketty’s work *Capital in the 21st century* will be analysed. Piketty’s most important statements for the process of wealth accumulation will be theoretically analysed and included in model 2 and 4. To understand Piketty’s mechanisms in the process of wealth accumulation the theoretical foundation will be discussed in the following part of this paper.

In Piketty’s analysis the capital/income ratio $\beta = \frac{K}{Y}$, measures the overall importance of capital in a society, but says nothing about social inequality within a country. Furthermore, $\beta$ is related to the share of income from capital in national income $\alpha$, which can be calculated with the formula $\alpha = r \times \beta$, where $r$ is the rate of return on capital. If now for example $\beta = 600\%$ and $r = 5\%$, then $\alpha = r \times \beta = 30\%$ and shows that the capital’s share in national income is 30 percent. Piketty states that this simple framework expresses a transparent relationship between the three most important concepts for analysing the capitalist system: $\beta$, $\alpha$ and $r$ [Piketty, 2014, p.51f].

According to Milanovic, Piketty’s key inequality relationship $r > g$ plays an important role in the production of social inequality. If the rate of return on capital, $r$, permanently is above the rate of growth of the economy, $g$, then $\alpha$ increases by definition and furthermore $\beta$ increases as well [Milanovic, 2013,
Kapeller states that the relationship \( r > g \), causes that the capital/income ratio, \( \beta \), increases. Therefore, the share of capital in national income, \( \alpha \), increases as well which leads to a redistribution from earned income to capital income. As a result the role of capital in the process of wealth accumulation increase and the relationship \( r > g \) can be interpreted as Piketty’s main cause of increasing social inequality [Kapeller, 2014, p.330f.]

Piketty tries to show the development of the rate of return to capital, \( r \), and the growth rate of world output, \( g \), in Figure 8. It is possible to observe that the rate of return to capital was nearly over the whole period above the growth rate. Only a concatenation of circumstances: wartime destruction, progressive tax policies and exceptional growth after World War II; created a historically unique situation where the growth rate of world output increased over the rate of return to capital. However, Piketty states that fiscal competition will cause that the rate of return to capital will increase over the growth rate of world output again [Piketty, 2014, p.356]. Therefore, social inequality will increase in the future and the process of wealth accumulation will be more affected through the influence of capital to the process of wealth accumulation.
Figure 10.10. After tax rate of return vs. growth rate at the world level, from Antiquity until 2100

The rate of return to capital (after tax and capital losses) fell below the growth rate during the 20th century, and may again surpass it in the 21st century. Sources and series: see piketty.pse.ens.fr/capital21c

Figure 10: The development of the rate of return to capital r and the growth rate of world output g (Piketty, 2014, p.356)

5.2 Model 2.1

To formalize Piketty’s assumption for the development of the rate of return to capital, r, and the growth rate of GDP, g, the models 2 and 3 will be modified by the relationship $r > g$. Therefore, the growth rate of GDP, $g$, was decreased from 3% in model 2 to 2.5% in model 2.1. Figure 9 shows compared to Figure 4 two differences.

First, the composition of the GDP has changed dramatically. In Model 2.1 capital income increased over the earned income significantly. After 100 rounds the share of capital income starts to take off and it is possible to observe the increasing influence of capital in the process of wealth accumulation. As a result, the importance of income out of work is decreasing and individuals with a high amount of wealth can accumulate capital easier.
Furthermore, in Figure 9 compared to Figure 4, Piketty’s Beta increased significantly, which highlights the increased importance of capital in the process of wealth accumulation. However, the indicators in Table 5 show that social inequality has decreased. The 20/20 and 10/10 ratio have decreased from 2.269 to 1.941, respectively from 3.199 to 2.564. Moreover, the gini-coefficient has decreased from 0.171 to 0.138. The decrease in social inequality can be explained by the fact that the relation \( r > g \) leads to a higher importance of capital in the process of wealth accumulation. In round 1 all individuals receive the same initial wealth of 10 and in model 2.1 the growth rate of earned income is not growing as fast as in model 2, which causes that the differences in wealth are created later on. As a result the Matthew-effect needs more time to be effective. Furthermore, the total, minimum and maximum wealth have decreased as well, which can be explained by the decrease in the growth rate from 3% to 2.5%.
5.3 Model 4.1

The implementation of the relationship $r > g$ in model 4 leads to a significant difference in the composition of the GDP. After round 100 the capital income takes off and surpasses the earned income clearly. In round 200 the GDP consists only out of capital income and earned income has decreased to a
<table>
<thead>
<tr>
<th></th>
<th>Model 2</th>
<th>Model 2.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>20/20 Ratio</td>
<td>2.269</td>
<td>1.941</td>
</tr>
<tr>
<td>10/10 Ratio</td>
<td>3.199</td>
<td>2.564</td>
</tr>
<tr>
<td>Gini-coefficient</td>
<td>0.171</td>
<td>0.138</td>
</tr>
<tr>
<td>Total wealth</td>
<td>16,101,785</td>
<td>10,117,305</td>
</tr>
<tr>
<td>Wealth min.</td>
<td>15,137</td>
<td>10,074</td>
</tr>
<tr>
<td>Wealth max.</td>
<td>101,876</td>
<td>63,070</td>
</tr>
</tbody>
</table>

Table 5: Own calculations

minimum. The decrease in the growth rate caused a decrease in the growth rate of earned income and the increasing importance of capital income caused that the share of earned income even decreased. Furthermore, in Figure 10 compared to Figure 5 Piketty’s Beta increased significantly, which highlights the increased importance of capital in the process of wealth accumulation. However, social inequality remained more or less the same, which can be observed by the indicators in Table 6. The implementation of r>g caused a small decrease in the 20/20, respectively 10/10, ratio from 3.506/5.924 in model 4 to 3.080/5.049 in model 4.1 and a small decrease of the gini-coefficient from 0.290 in model 4 to 0.257 in model 4.1.
Figure 12: Own Illustration

<table>
<thead>
<tr>
<th></th>
<th>Model 4</th>
<th>Model 4.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>20/20 ratio</td>
<td>3.506</td>
<td>3.080</td>
</tr>
<tr>
<td>10/10 ratio</td>
<td>5.924</td>
<td>5.049</td>
</tr>
<tr>
<td>Gini-coefficient</td>
<td>0.290</td>
<td>0.257</td>
</tr>
<tr>
<td>Total wealth</td>
<td>18,473,695</td>
<td>11,473,494</td>
</tr>
<tr>
<td>Wealth min.</td>
<td>14,602</td>
<td>7,984</td>
</tr>
<tr>
<td>Wealth max.</td>
<td>315,320</td>
<td>221,493</td>
</tr>
</tbody>
</table>

Table 6: Own calculations
6 Résumé

The theory of the Matthew-effect showed that due to cumulative advantages initial differences get larger in the future. By an example of Rigney, who showed that owing to interest rates small advantages at the beginning of a process lead to larger advantages over time, the foundation for a formalization of a direct network effect of wealth accumulation was found. Furthermore, Bourdieu’s capital theory showed that social and cultural capital influence the ability to accumulate economic capital and that it is necessary to take these two forms of capital into account, when the process of wealth accumulation is analysed. Economic and cultural/social capital are two compatible systems, which are causing positive feedback effects to each other and an indirect network effect was found.

Next Pareto’s theory of society was analysed to find a second indirect network effect. It became obvious that power and wealth are compatible and favour each other. According to the analysis of Pareto’s theory of society social elites have both, power and wealth. Therefore, it was supposed that social elites build legal systems in form of shadow banks, which guarantee higher interest rates for them and as a result reproduce social inequality.

The Matthew-effect, social/cultural capital and power in the form of shadow banks were formalized and simulated in model 2, 3 and 4. Owing to the implementation of direct and indirect network effects the process of wealth accumulation has changed. Table 4 illustrates that all indicators have increased. Therefore, it becomes clear that due to direct and indirect network effects social inequality will increase over time. Moreover, a small social elite will be created which can increase their wealth over time caused by the increasing importance of capital income in the process of wealth accumulation. As a result the rest of society which have lower capital income caused by their lower wealth stocks that depend more on earned income. Caused by the increasing share of capital income compared to the earned income of the GDP the growth rate of earned income is decreasing. Therefore, cumulative advantages cause that wealth leads to more wealth and it can be observed that the rest of society cannot catch up. As a result, the process of wealth
accumulation, in consideration of the assumptions of the developed models, leads to a lock-in where poor people remain poor and rich people remain rich. Due to the analysis of Piketty’s Capital in the 21st century the relation $r > g$ was implemented in model 2 and 4. It became obvious that caused by the decreased growth rate, the earned income became less important in the process of wealth accumulation. Moreover, the role of capital income increased and wealth can be easier obtained through a high amount of capital than by work. This leads to the conclusion that direct and indirect networks effects influence the process of wealth accumulation and need to be taken into account more seriously in the current debate about social inequality.
7 Literature


Rigney, Daniel (2010): The Matthew effect - How advantage begets further
advantage. New York: Columbia University Press

library(ineq)

# Model 1

n=500  # Number of Persons
m=200  # Number of Rounds
y=matrix(NA,m+1,n)  # wealth-matrix
ec=matrix(NA,m+1,n)  # income-matrix
c=matrix(NA,m+1,n)  # consumption-matrix
Bip=NA  # GDP
v=matrix(0,m+1,n)

m=NA

nw[1]=10  # mean of wealth in round 1

for(j in 1:m){

  for(i in 1:n){
    e[1,i]=rnorm(1,5,1)  #working income
    y[1,i]=10  #starting wealth
    c[j,i]=0.9*e[j,i]  #consumption
    y[j+1,i]=y[j,i]+c[j,i]-c[j,i]  #calculation of wealth
    e[j+1,i]=e[j,i]  # income
  }

  nw[j]=mean(y[j,])
  Bip[1]=sum(e[1,])+sum(v[1,])
  Bip[j+1]=Bip[j]
}

Bip
e
y

s=y[m,]
s = sort(s)
quantile(s)

###
# Plot
###
plot(s, type='l', ylim=c(min(s), max(s)), main='Distribution of wealth after 200 rounds', ylab='Wealth', xlab='Individuals')

#################
# Indicators
#################

# 20:20 Ratio
sum(s[401:500])/sum(s[1:100])

# 10:10 Ratio
sum(s[451:500])/sum(s[1:50])

# Gini-Koeffizient
ineq(s, type='Gini')

# wealth in round 200
sum(s)

# max wealth
max(s)

# min wealth
min(s)

######################
# Model 2
######################
n = 500
\( m = 200 \)
\( y = \text{matrix}(0, m+1, n) \)
\( e = \text{matrix}(0, m+1, n) \)
\( c = \text{matrix}(0, m+1, n) \)

\( \text{Bip} = \text{NA} \) \hspace{1cm} \# GDP
\( g = 0.03 \) \hspace{1cm} \# growth of GDP
\( \text{ek} = 0.00 \) \hspace{1cm} \# rate of growth earned income round 1

\( r = \text{matrix}(0, m+1, n) \)
\( v = \text{matrix}(0, m+1, n) \)

\begin{verbatim}
for (j in 1:m) {

  mw[1] = 10

  for (i in 1:n) {
    e[1,i] = rnorm(1, 5, 1) \hspace{1cm} \# earned income
    y[1,i] = 10
    r[1,i] = rnorm(1, 3, 0.1)/100 \hspace{1cm} \# return on capital
    r[j,i] = rnorm(1, 3, 0.1)/100
    v[j,i] = y[j,i] * r[j,i] \hspace{1cm} \# capital income
    c[1,i] = 0.9 * e[1,i] \hspace{1cm} \# consumption

    \# savings rate
    if (y[j,i] >= 1.2 * mw[j]) { e[j,i] = e[j,i] * 0.75 }
    else if (y[j,i] >= 1.5 * mw[j]) { c[j,i] = c[j,i] * 0.65 }
    else { c[j,i] = e[j,i] * 0.9 }

    \# calculation earned income
    e[j+1,i] = e[j,i] * (1 + ek[j])

    \# calculation capital income
    y[j+1,i] = y[j,i] + e[j,i] - c[j,i] + v[j,i]
  }

  mw[j+1] = mean(y[j+1,]) \hspace{1cm} \# mean of wealth
  Bip[1] = sum(e[1,]) + sum(v[1,]) \hspace{1cm} \# consumption GDP

}\end{verbatim}
Bip[ j+1]=Bip[ j]*(1+g)  # growth of GDP

# growth rate of earned income
ek[ j+1]=(Bip[ j+1]-(sum(v[ j, ])))/(sum(e[ j, ]))-1

}

Bip
ey

s=y[m,]
s=sort(s)
quantile(s)

#####
# Plot
#####

par(mfrow=c(2,2))
plot(s, type='l', ylim=c(min(s),max(s)), main='Distribution of wealth after 200 rounds', ylab='wealth', xlab='Individuals')
plot(Bip, type='l', main='', xlab='Rounds', ylab='')
t=NA
t1=NA
e1=NA
ev1=NA

for(i in 1:m) {
   e1[i]=sum(e[i,1:500])
}

lines(e1,col='red')

for(i in 1:m) {
   v1[i]=sum(v[i,1:500])
}
\texttt{lines(v1,col='green')}  
\texttt{legend('topleft', c('GDP', 'earned income', 'capital income'),}  
\texttt{col=c('black', 'red', 'green'), pch=c(19,19,19))}  

\texttt{for(i in 1:m) {}  
\hspace{1em} t1[i]=sum(y[i,1:500])}  

\texttt{)}  
\texttt{plot(t1, type='line', main='total wealth', xlab='Rounds', ylab='total wealth')}  
\texttt{plot(sort(t1/Bip), type='l', main='Piketty's Beta', ylab='Beta', xlab='Rounds')}  

\texttt{################}  
\texttt{#Indicators}  
\texttt{################}  

\texttt{#20:20}  
\texttt{sum(s[401:500])/sum(s[1:100])}  

\texttt{#10:10}  
\texttt{sum(s[451:500])/sum(s[1:50])}  

\texttt{#Gini}  
\texttt{ineq(s, type='Gini')}  

\texttt{#wealth in round 200}  
\texttt{sum(s)}  

\texttt{#max wealth}  
\texttt{max(s)}  

\texttt{#min wealth}  
\texttt{min(s)}  

\texttt{###################}  
\texttt{# Model 3}
n=500
m=200
y=matrix(0,m+1,n)
e=matrix(0,m+1,n)
c=matrix(0,m+1,n)
Bip=NA
g=0.03
ek=0.00
r=matrix(0,m+1,n)
v=matrix(0,m+1,n)

for(j in 1:m){

  mw[1]=10

  for(i in 1:n){
    e[1,i]=rnorm(1,5,1)
y[1,i]=10
    r[1,i]=rnorm(1,3,0.1)/100
    r[j,i]=rnorm(1,3,0.1)/100
    v[j,i]=y[j,i]*r[j,i]
c[j,i]=0.9*e[1,i]

    # savings rate
    if(y[j,i]>=1.2*mw[j]) {c[j,i]=e[j,i]*0.75}
    else if(y[j,i]>=1.5*mw[j]) {c[j,i]=e[j,i]*0.65}
    else {c[j,i]=e[j,i]*0.9}

    # calculation earned income
    e[j+1,i]=e[j,i]*(1+ek[j])

    # simulation of cultural and social capital
    if(y[j,i]>1.1*mw[j]) {e[j,i]=e[j,i]*1.2}

    # calculation wealth
    y[j+1,i]=y[j,i]+e[j,i]-c[j,i]+v[j,i]
mw[j+1]=mean(y[j+1,])
Bip[1]=sum(e[1,])+sum(v[1,])
Bip[j+1]=Bip[j]*(1+g)

ek[j+1]=(Bip[j+1]-(sum(v[j,]))/(sum(e[j,]))-1

Bip
e
y

s=y[m,]
s=sort(s)
quantile(s)

###
#Plot
###
par(mfrow=c(2,2))
plot(s, type='l', ylim=c(min(s),max(s)), main='Distribution of wealth after 200 rounds', ylab='wealth', xlab='Individuals')
plot(Bip, type='l', main='', xlab='Rounds', ylab='')
t=NA
t1=NA
e1=NA
v1=NA

for(i in 1:m) {

e1[i]=sum(e[i,1:500])

}

lines(e1,col='red')
for(i in 1:m) {
    v1[i]=sum(v[i,1:500])
}

lines(v1,col='green')
legend('topleft', c('GDP', 'earned income', 'capital income'),
    col=c('black', 'red', 'green'), pch=c(19,19,19))

for(i in 1:m) {
    t1[i]=sum(y[i,1:500])
}

plot(t1, type='line', main='total wealth', xlab='Rounds', ylab='total wealth')

plot(sort(t1/Bip), type='l', main='Piketty's Beta', ylab='Beta',
    xlab='Rounds')

####################################
#Indicators
####################################

#20:20
sum(s[401:500])/sum(s[1:100])

#10:10
sum(s[451:500])/sum(s[1:50])

#Gini
ineq(s,type='Gini')

#wealth in round 200
sum(s)

#max wealth
max(s)
#min wealth
min(s)

#############################
# Model 4
#############################

n=500
m=200
y=matrix(0,m+1,n)
e=matrix(0,m+1,n)
c=matrix(0,m+1,n)
Bip=NA
g=0.03
eK=0.00
r=matrix(0,m+1,n)
v=matrix(0,m+1,n)
rr=matrix(0,m+1,n)
a=NA

for(j in 1:m){
    mw[1]=10
    for(i in 1:n){
        e[1,i]=rnorm(1,5,1)
y[1,i]=10
        r[1,i]=rnorm(1,3,0.1)/100
        r[j+1,i]=rnorm(1,3,0.1)/100
        v[j,i]=y[j,i]*r[j,i]
c[1,i]=0.9*e[1,i]
    }

    #savings rate
    if(y[j,i]>=1.2*mw[j]){c[j,i]=c[j,i]*0.75}
    else if(y[j,i]>=1.5*mw[j]){c[j,i]=e[j,i]*0.65}
    else{c[j,i]=e[j,i]*0.9}
#calculation earned income
e_{j+1,i}=e_{j,i}*(1+e_k[j])

#simulation shadow banks
if(y[j,i]>1.5*mw[j]) {r[j+1,i]=r[j+1,i]+0.01}
}
#else {rr[j,i]=r[j,i]}
#calculation wealth
y[j+1,i]=y[j,i]+e[j,i]-c[j,i]+v[j,i]

#v[j+1,i]=y[j,i]*r[j,i]

mw[j+1]=mean(y[j+1,i])
Bip[1]=sum(e[1,i])+sum(v[1,i])
Bip[j+1]=Bip[j]*(1+g)
e_k[j+1]=(Bip[j+1]-(sum(v[j,i])))/(sum(e[j,i]-1)

Bip
e
y

s=y[m,i]
s=sort(s)
quantile(s)

####
#Plot
####
par(mfrow=c(2,2))
plot(s, type='l', ylim=c(min(s),max(s)), main='Distribution of wealth after 200 rounds', ylab='wealth', xlab='Individuals')
plot(Bip, type='l', main='', xlab='Rounds', ylab='')
t=NA
t1=NA
e1=NA
v1=NA

for(i in 1:m) {
  e1[i]=sum(e[i,1:500])
}
lines(e1,col='red')

for(i in 1:m) {
  v1[i]=sum(v[i,1:500])
}
lines(v1,col='green')
legend('topleft', c('GDP','earned income', 'capital income'),
  col=c('black', 'red', 'green'), pch=c(19,19,19))

for(i in 1:m) {
  t1[i]=sum(y[i,1:500])
}

plot(t1, type='line', main='total wealth', xlab='Rounds', ylab='total wealth')
plot(sort(t1/Bip), type='l', main='Piketty’s Beta', ylab='Beta',
  xlab='Rounds')

#################
#Indicators
#################

#20:20
sum(s[401:500])/sum(s[1:100])

#10:10
\[ \frac{\text{sum}(s[451:500])}{\text{sum}(s[1:50])} \]

#Gini
ineq(s, type="Gini")

#wealth in round 200
sum(s)

#max wealth
max(s)

#min wealth
min(s)

############################################################
# Modellvariante 2.1
############################################################

n=500
m=200
y=matrix(0,m+1,n)
e=matrix(0,m+1,n)
c=matrix(0,m+1,n)
Bip=NA
g=.025
ek=.000
r=matrix(0,m+1,n)
v=matrix(0,m+1,n)

for(j in 1:m){

    msw[1]=10

    for(i in 1:n){
        e[1,i]=rnorm(1,5,1)
y[1,i]=10
        r[1,i]=rnorm(1,3,0.1)/100
        r[j,i]=rnorm(1,3,0.1)/100
    }
}
\[ v[j, i] = y[j, i] \times r[j, i] \]
\[ c[1, i] = 0.9 \times e[1, i] \]

```
# savings rate
if (y[j, i] \geq 1.2 \times \text{mw}[j]) \{ c[j, i] = e[j, i] \times 0.75 \}
else if (y[j, i] \geq 1.5 \times \text{mw}[j]) \{ c[j, i] = e[j, i] \times 0.65 \}
else \{ c[j, i] = e[j, i] \times 0.9 \} 

# calculation earned income
\[ e[j+1, i] = e[j, i] \times (1 + e_k[j]) \]

# calculation wealth
\[ y[j+1, i] = y[j, i] + e[j, i] - c[j, i] + v[j, i] \]
```

\[ \text{mw}[j+1] = \text{mean}(y[j+1, :]) \]
\[ B[j] = \text{sum}(e[1, :]) + \text{sum}(v[1, :]) \]
\[ B[j+1] = B[j] \times (1 + g) \]
\[ e_k[j+1] = (B[j+1] - (\text{sum}(v[j, :])) / (\text{sum}(e[j, :])) - 1 \]

```
B[j] = e[j] + y[j, :]

s = y[m, :]
s = \text{sort}(s)
quartile(s)
```

```r
par(mfrow=c(2,2))
```
plot(s, type="l", ylim=c(min(s), max(s)), main="Vermögensverteilung in Runde 200", ylab="Vermögen", xlab="Individuen")

plot(Bip, type="l", main="", xlab="Runden", ylab="")

t=NA
t1=NA
e1=NA
v1=NA

for(i in 1:m) {
    e1[i]=sum(e[i,1:500])
}

lines(e1,col='red')

for(i in 1:m) {
    v1[i]=sum(v[i,1:500])
}

lines(v1,col='green')
legend('topleft', c('GDP', 'earned income', 'capital income'), col=c('black', "red", "green"), pch=c(19,19,19))

for(i in 1:m) {
    t1[i]=sum(y[i,1:500])
}

plot(t1, type="line", main="total wealth", xlab="Rounds", ylab="total wealth")
plot(sort(t1/Bip), type='l', main='Piketty's Beta', ylab='Beta', xlab='Rounds')
### Indicators

#### 20:20
\[
\text{sum}(s[401:500])/\text{sum}(s[1:100])
\]

#### 10:10
\[
\text{sum}(s[451:500])/\text{sum}(s[1:50])
\]

#### Gini
\[
\text{ineq}(s, \text{type}='\text{Gini}')
\]

#### Wealth in round 200
\[
\text{sum}(s)
\]

#### Max wealth
\[
\text{max}(s)
\]

#### Min wealth
\[
\text{min}(s)
\]

### Modellvariante 4.1

n=500
m=200
\[
y=\text{matrix}(0,m+1,n)
\]
\[
e=\text{matrix}(0,m+1,n)
\]
\[
c=\text{matrix}(0,m+1,n)
\]
Bip=NA
g=0.025
ek=0.00
\[
r=\text{matrix}(0,m+1,n)
\]
\[
v=\text{matrix}(0,m+1,n)
\]
\[
rr=\text{matrix}(0,m+1,n)
\]
a=NA
for (j in 1:m) {

    mw[1] = 10

    for (i in 1:n) {
        e[1, i] = rnorm(1, 5, 1)
        y[1, i] = 10
        r[1, i] = rnorm(1, 3, 0.1) / 100
        r[j+1, i] = rnorm(1, 3, 0.1) / 100
        v[j, i] = y[j, i] * r[j, i]
        c[1, i] = 0.9 * e[1, i]

        # savings rate
        if (y[j, i] > 1.2 * mw[j]) { c[j, i] = e[j, i] * 0.75 }
        else if (y[j, i] > -1.5 * mw[j]) { c[j, i] = c[j, i] * 0.65 }
        else { c[j, i] = e[j, i] * 0.9 }

        # calculation earned income
        e[j + 1, i] = e[j, i] * (1 + ek[j])

        # simulation shadow banks
        if (y[j, i] > 1.5 * mw[j]) { r[j + 1, i] = r[j + 1, i] + 0.01 }
    }

    }  # else { rr[j, i] = r[j, i] }

    # calculation wealth
    y[j + 1, i] = y[j, i] + e[j, i] - c[j, i] + v[j, i]

    # v[j + 1, i] = y[j, i] * r[j, i] 
}

mw[j + 1] = mean(y[j + 1, ])
Bip[1] = sum(e[1, ] + sum(v[1, ])
Bip[j + 1] = Bip[j] * (1 + g)
ek[j + 1] = (Bip[j + 1] - (sum(v[j, ]))) / (sum(e[j, ])) - 1
}

Bip
e
y

s=y[m,]
s=sort(s)
quantile(s)

###
#Plot
###

par(mfrow=c(2,2))
plot(s, type="l", ylim=c(min(s),max(s)), main='Distribution of wealth after 200 rounds', ylab='wealth', xlab='Individuals')
plot(Bip, type="l", main='', xlab='Rounds', ylab='')
t=NA
t1=NA
ev=NA
v1=NA

for(i in 1:m) {
    e1[i]=sum(e[i,1:500])
}
lines(e1,col='red')

for(i in 1:m) {
    v1[i]=sum(v[i,1:500])
}
lines(v1,col='green')
legend('topleft', c('GDP','earned income','capital income'),
       col=c('black','red','green'), pch=c(19,19,19))

for(i in 1:m) {
    t1[i]=sum(y[i,1:500])
}
```r
}

plot(t1, type="line", main="total income", xlab="Rounds", ylab="total income")
plot(sort(t1/Bip), type='l', main='Piketty’s Beta', ylab='Beta', xlab='Rounds')

#################################
#Indicators
#################################

#20:20
sum(s[401:500])/sum(s[1:100])

#10:10
sum(s[451:500])/sum(s[1:50])

#Gini
ineq(s, type='Gini')

#wealth in round 200
sum(s)

#max wealth
max(s)

#min wealth
min(s)
```